matter is one of the central open problems of physics whose solution may well require another radical change in the physicist's conception of nature. "I do not believe that a real understanding of the nature of elementary particles can ever be achieved without a simultaneous deeper understanding of the nature of spacetime itself", says R. Penrose [13] in his profound lectures on the structure of spacetime.

The purpose of this review is to describe the spacetime models which are currently important in physics, namely the special (flat) and general (curved) nonrelativistic spacetimes and the corresponding relativistic ones; to discuss reasons for their adoption or rejection for certain domains of experience; to sketch some recent work related to spacetime structure; and to hint at weaknesses and possible future developments. The emphasis is placed on the present understanding of spacetime structure; the historical development of ideas will be indicated only. Accordingly, even old ideas will be described in the language of present-day mathematics and physics.

## 2. Nonrelativistic theories of spacetime

"The objects of our perception invariably include places and times in combination. Nobody has ever noticed a place except at a time, or a time except at a place" (Minkowski [4]). The primary extensive medium in which physical processes are imagined to occur (or, more abstractly, the common domain of definition of those fields which represent – at least at the classical, macroscopic level – the observable quantities) is therefore taken to be spacetime, a set $M$ whose elements $p, q, \ldots$ are called events. Whether and how $M$ can be decomposed into a 3-dimensional space and a 1-dimensional time is already a question about the structure of $M$ whose answer is subject to empirical test.

According to Newton, there is an absolute time and an absolute space, recognizable by observations of mechanical phenomena such as the curvature of the surface of water in an "absolutely rotating" pail. This means that for any two events $p, q$ it is regarded as objectively decidable whether they are simultaneous and also whether they occur at the same place. Hence the set $M$ of events is the cartesian product of the set $T$ of all instants (of time) and the set $S$ of all space points,

$$M = S \times T. \tag{2.1}$$

Moreover, $S$ is assumed to be a Euclidean 3-space whose metric $dl^2$ is measurable by means of rulers, and $T$ is taken to be a one-dimensional Euclidean space whose natural coordinate $t$ (defined up to linear transformations $t \to at + b$) is given by standard clocks. This Newtonian spacetime is illustrated in Fig. 1.

The stratification of $M$ given by the maximal subsets of simultaneous events can be interpreted as the causal structure of $M$, and idea apparently due to Leibniz [14]. The hyperplane $t = t(e)$ passing through an event $e$ separates the causal future, or the domain of influence ($t > t(e)$) of $e$, from its causal past. The circumstance that future and past of $e$ have a common boundary, the present, expresses the hypothesis implicit in New-

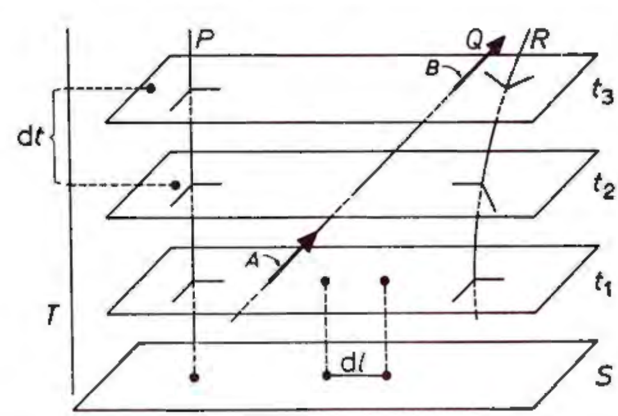


Fig. 1. Newtonian Spacetime. Particle $P$ is at absolute rest and carries non-rotating axes, $Q$ moves uniformly, and $R$ is accelerated and carries rotating axes. $A$ and $B$ are parallel.

tonian Physics that there are arbitrarily fast signals, realizable by means of (strictly or arbitrarily nearly) rigid bodies or instantaneous action of a distance.

The stratification is also the formal counterpart of the ontological idea that the external world evolves in time: not only for any particular observer, but objectively the present state of the world is supposed to consist of the distribution of matter in the hyperplane "now" existing, and the succession of the configurations of bodies in these hyperplanes represents the history of the material universe.

The group of transformations which preserve the structure of Newtonian spacetime –the product structure (2.1), and spatial and temporal congruence– is the direct product of the group of dilations, rotations and translations of $S$ with the affine group of $T$; following Weyl [15] we call it the elementary group $\mathfrak{E}$.

Whereas the spatial and temporal metrics of $M$ have a sound empirical foundation and the corresponding causal structure was an acceptable idealization as long as there was no clear evidence against instantaneous transmission of influences, the postulated absolute standards of rest and of no rotation have been rightly questioned by Berkeley, Huyghens and Leibniz [16] on the basis of the relativity of motion. If one drops these last two assumptions, one is led to (say) Leibniz's spacetime which is illustrated in Fig. 2.

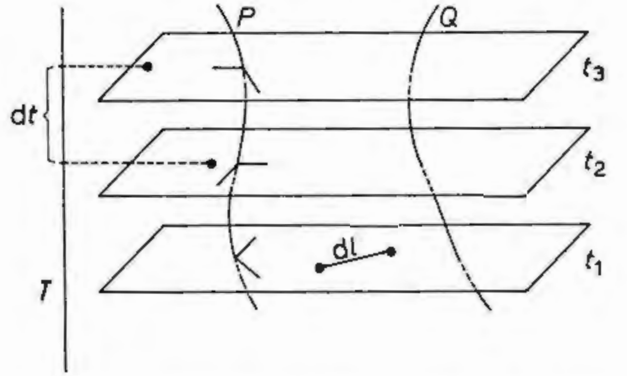


Fig. 2. Leibniz's Spacetime. There are no preferred motions (no "straight" lines), and there is no parallelism except the one within a single stratum $t$ = const.

Leibniz's spacetime, the spacetime of nonrelativistic kinematics, has less structure than Newton's; in it there are neither preferred motions nor preferred "nonrotating" spatial axes. Whereas Newtonian spacetime is an affine 4-space –since it is the product of Euclidean, hence affine spaces– Leibniz's spacetime has no affine structure; i.e. in the former one can speak of parallel 4-vectors, in the latter that is meaningless. Accordingly, the group of automorphisms of Leibniz's spacetime, called the kinematical group $\mathfrak{K}$, is much larger than the elementary group $\mathfrak{E}$. $\mathfrak{E}$ is a 9-dimensional Lie group, $\mathfrak{K}$ is not a Lie group since its elements require for their specification not only 3 parameters but also 6 arbitrary real functions of time (an angular velocity and a translation velocity).

Newtonian spacetime is obtained from Leibniz's spacetime by adding to the causal and metric structures of the latter the fibration which defines the state of absolute rest.

It is clear that a decision between these two spacetime models requires dynamical arguments. Newton's famous discussion of the spinning water-bucket (or Foucault's pendulum) can serve to justify the assumption that, dynamically, rotation has an absolute meaning, so that one aspect of the Newtonian spacetime structure, the parallel transport of spatial axes along timelike worldlines, is physically acceptable. In this respect dynamical facts decide in favour of Newton and against his relativistic opponents Huyghens and Leibniz. On the other hand, to identify in nature the state of absolute rest, Newton resorts to the statement "that the centre of the system of the world is at rest", and this assertion has no observationally testable content.

Thus, both Newton and Leibniz are correct in their mutual criticisms, but the spacetime geometries corresponding to their positions are both dynamically inadequate. Leibniz's kinematical spacetime has not enough, Newton's dinamical spacetime has too much structure [29] (Equation (2.1)! ). It is a remarkable historical fact that the resulting lack of clarity in the foundations of dynamics, although it was felt by many scientists, notably Euler, persisted until Lange [17] in 1885 recognized that what is needed besides the causal and metric structures is (in modern terms) the assumption that spacetime is an affine 4-space whose timelike straight lines (i.e. those not contained in a hyperplane $t$ = const.) represent free motions. This axiom is a precise formulation of the law of inertia (see Fig. 3), which emphasizes its intrinsic, coordinate-independent content [18].

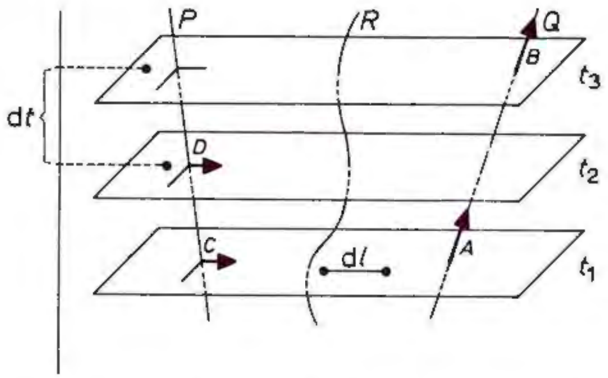


Fig. 3. The spacetime of classical dynamics. Uniform motion, exemplified by $P$ and $Q$, is considered as absolutely preferred against accelerated motion, $R$; but no absolute rest (verticals) is defined. Parallelity of 4-vectors is meaningful; $A \parallel B$, $C \parallel D$.

It has often been argued that Newton's first law should not be postulated as a separate axiom since it is implied by the second law. That is of course true, but far more important than this trivial implication is that the law of inertia alone serves to define the affine structure of spacetime. The subsequent laws of dynamics presuppose that structure but do not restrict or enrich the spacetime geometry further.

Accepting the law of inertia one obtains the well-known spacetime of classical dynamics which we call "special" nonrelativistic spacetime, for reasons to be explained later. Its group of automorphisms, the Galilei group $\mathfrak{G}$, is intermediate between $\mathfrak{E}$ and $\mathfrak{K}$,

$$\mathfrak{E} \subset \mathfrak{G} \subset \mathfrak{K}. \tag{2.2}$$

The three groups $\mathfrak{E}$, $\mathfrak{G}$, $\mathfrak{K}$ characterize the geometrical or, perhaps better, chronogeometrical structure of the corresponding spacetimes precisely in the sense of Klein's Erlanger programme (1872) [19].

$\mathfrak{G}$ is not only the symmetry group of the spacetime of classical nonrelativistic dynamics, but also the invariance group of the laws of mechanics which govern isolated systems of particles (or extended bodies). This statement is the essential content of the Galilean principle of relativity, which can be rephrased as the equivalence of all inertial reference frames for the description of dynamical phenomena. Within the spacetime so obtained also the nonrelativistic quantum theories of particles and fields can be formulated. The structure of $\mathfrak{G}$ plays, via its unitary ray representations, an essential role in constructing from first principles the quantum theory of free particles [20].

At this stage we depart from the (anyhow largely simplified) course of history. Whereas in fact the next two important steps in the evolution of spacetime concepts were taken by Einstein in 1905 and 1915, we should like to describe here first a natural extension of the special nonrelativistic spacetime due to Cartan [21] and Friedrichs [22] and elaborated further by Havas [23] and Trautman [24]. This theory was actually formulated only after and under the influence of Einstein's special and general theories, but from a systematic point of view it should be considered prior to these theories. Its merit is to show already at the nonrelativistic level that a satisfactory incorporation of gravity into the system of spacetime geometry and mechanics requires, because of the well established universal proportionality of inertial and (passive) gravitational mass, a change in the affine structure of spacetime. This step is independent of the relativization of time, since the phenomena in question do not necessarily involve large speeds or energies and thus are not relativistic.

Cartan's theory can be motivated by the following considerations [25]. The transition from kinematics to dynamics as sketched above consists in singling out a preferred class of motions the members of which define everywhere a standard of "no acceleration"; the law which characterizes these motions is called the first law of dynamics. Once that has been done —and only then— forces are introduced via Newton's second law of dynamics; the acceleration of an arbitrary motion is to be judged relative to a preferred motion with the same instantaneous velocity passing (nearly) through the same event as the arbitrary motion. (This formulation presupposes a metric of spacetime, but avoids the use of dynamically preferred frames of reference and is purely local; it is meaningful at the level of the group $\mathfrak{K}$.) The traditional way of carrying out this general programme is to choose as the first law the law of inertia which leads to the spacetime geometry and dynamics belonging to the group $\mathfrak{G}$, as reviewed above. There is a grave objection to this procedure, however. Because of the experimentally extremely well established composition-independence of the ratio passive gravitational mass/inertial mass (see Dicke et al. [5] and Braginski et al.[5]) of macroscopic test bodies the actually available, unique candidate for the preferred class of motions is the class of free falls of (neutral, spherically symmetric, non-rotating) test bodies. These motions do not permit and observationally meaningful distinction between inertial forces and gravitational forces (and neither do any other known phenomena), and they do not satisfy the law of inertia since they exhibit relative accelerations. This insight suggests strongly to abandon the law of inertia and to reconstruct dynamics by taking as the first law the following characterization of free falls: With respect to suitable (so-called "non-rotating") frames of reference, free falls obey the law

$$\ddot{\mathbf{x}} = -\nabla\phi, \tag{2.3}$$

in which $\phi\ (\mathbf{x}, t)$ is a frame-dependent real function called the gravitational potential relatively to that frame. This formulation is not completely satisfactory because it describes the structure provided by the class of free fall motions not directly, but with the help of preferred frames which are themselves defined by the free falls only. However, Equation (2.3) can be rewritten in the form

$$\begin{gathered}\frac{\mathrm{d}^2x^a}{\mathrm{d}t^2} + \Gamma^a_{bc}\frac{\mathrm{d}x^b}{\mathrm{d}t}\frac{\mathrm{d}x^c}{\mathrm{d}t} = 0,\\ \Gamma^\lambda_{44} = \phi_{,\lambda}(a, b, c = 1, ..., 4;\ \ t = x^4;\ \ \lambda = 1, 2, 3).\end{gathered} \tag{2.4}$$

Hence, there exists on spacetime a unique symmetric, linear connection $\Gamma$ the geodesics of which represent the world lines of free fall. This statement is the core of Cartan's theory. The connection $\Gamma$ together with the absolute time $t$ and the spatial metric $\mathrm{d}l^2$ can be characterized by some axioms [24] which ensure that in suitable coordinates $\Gamma^a_{bc}$ can be expressed as in (2.4); Poisson's equation relating the gravitational field to its material sources turns out to be expressible as a relation between the contracted curvature tensor of $\Gamma$, the density of matter and the gradient of $t$; and the second law of dynamics can be formulated as in general relativity theory in terms of covariant derivatives with respect to $\Gamma$.

Whereas the parallel displacement of 4-vectors defined by $\Gamma$ is path-independent if applied to spacelike vectors (which, by definition, are tangent to the space sections and are nothing but the familiar 3-vectors), it

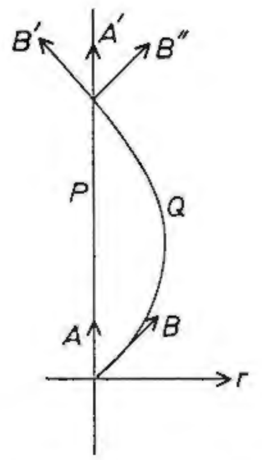


Fig. 4. A tunnel $T$, drilled along a diameter through a massive spherical body $B_0$, contains two freely falling particles $P$, $Q$. $P$ rests at the centre of $T$ and $B_0$, and $Q$ oscillates in $T$. The spacetime diagram shows the world lines of $P$ and $Q$. With respect to the gravitational-inertial connection $\Gamma$ defined in the text, one has obvionsly: Parallel transport along $P$ transfers $A$ to $A'$, $B$ to $B''$, and parallel transport along $Q$ transforms $B$ into $B'$. Since $B' \neq B''$, the $\Gamma$-parallel transport is not integrable. $P$ and $Q$ form a geodesic "diangle".

is in general path-dependent if applied to timelike vectors, as can be inferred from the curvature tensor associated with (2.4) or from dynamical thought-experiments like the one shown in Fig. 4 due to Heckmann and Schücking [26]. The non-integrability of the gravitational-inertial connection implies taht geodesics are curved relatively to each other, or in physical terms that freely falling particles exhibit relative accelerations. The quantitative measure of this spacetime curvature or gravitational tidal field is the curvature tensor associated with $\Gamma$.

The elementary mathematical fact that one can always introduce new coordinates $(x, t)$ such that, at an event or even along a free-fall-worldline, the components of $\Gamma$ vanish, corresponds to the physical fact, recognized by Einstein, that locally any gravitational field can be "transformed away" (elevator experiment).

Whereas formally the local laws of Cartan's theory of spacetime, gravity and dynamics (as completed by the authors mentioned above), if expressed with respect to non-rotating coordinate-frames, do not differ from those of the standard Newtonian theory as given in the textbooks, conceptually it embodies an important advance by denying the separate existence of an integrable affine connection representing the inertial field and a vector field representing gravitation, and introducing instead of these two structures a single non-integrable connection representing both inertia and gravity. An empirically unjustifiable, fictitious distinction has thereby been removed, and the true nature of gravity as a connection has been recognized. The introduction of this concept (which is due to Levi-Civita (1917) and Schouten (1918) [27]) into physics, which resulted from the work of Einstein [6], Weyl [15], Cartan [21] and others is comparable to the introduction of vector fields for the description of electromagnetism by Maxwell. Connections, besides vector and spinor fields, form another type of mathematical entities suitable to represent physical objects, an insight exploited in the theory of gauge fields [28].

A spacetime $M$ with a nonrelativistic metric $(t, \mathrm{d}l^2)$ and a connection $\Gamma$ according to (2.4) will be called a "general-nonrelativistic" spacetime. In it, there are no exact, global inertial frames, but only local inertial frames, and these exhibit relative translational, though no rotational accelerations. The group $\mathfrak{N}$ relating non-rotating coordinate systems, consisting of those transformations which leave $(t, \mathrm{d}l^2)$ and the form of the law (2.3) unchanged, is larger than the Galilean group $\mathfrak{G}$, but smaller than the kinematical group $\mathfrak{K}$. $\mathfrak{N}$ contains arbitrarily time-dependent translations, but only time-independent rotations. We may extend (2.2) to

$$\mathfrak{E} \subset \mathfrak{G} \subset \mathfrak{N} \subset \mathfrak{K}, \tag{2.5}$$

a relation which indicates in a condensed form the evolution of the spacetime concepts at the nonrelativistic level. Whereas the transition from $\mathfrak{E}$ to $\mathfrak{G}$ represents the preliminary compromise between the absolutist Newton ($\mathfrak{E}$) and the relativist Leibniz ($\mathfrak{K}$), the step from $\mathfrak{G}$ to $\mathfrak{N}$ –or from a flat to a curved connection– is a (somewhat delayed) response to Mach's criticism [30] of the unfounded distinction between inertia and gravity.

All of nonrelativistic physics including quantum mechanics can be reformulated without difficulty within the framework of general-nonrelativistic spacetime [31]. (Thus, e.g., the change of the gravitational potential associated with a transformation of $\mathfrak{N}$ is accompanied by a phase change of a Schrödinger wave function to ensure form-invariance of the Schrödinger equation.) All the non-gravitational local laws have, in local inertial frames, the same form as in the gravity-free, special spacetime. Thus Einstein's strong principle of equivalence [32] is incorporated satisfactorily (as far as slow motion, low energy phenomena are concerned) into nonrelativistic physics. The "special" theory based on $\mathfrak{G}$ now appears as a local approximation to the "general" theory, valid as long as inhomogeneities of the gravitational field can be neglected.

One principal advantage of the generalized version of Newtonian mechanics is that it permits the treatment of unbounded and, in particular, spatially homogeneous selfgravitating systems as used in cosmology [33].

In the transition from the special to the general nonrelativistic spacetime the status of the connection has been changed from that of an absolute element [34], given once and for all, to a dynamical quantity depending on the physical state. The metric, however, is still treated as an absolute element. This is possible since a Galilean metric ($t$, $dl^2$) does not determine a unique connection (2.4), in contrast to the situation in relativity theory.

## 3. Relativistic theories of spacetime

Whereas nonrelativistic physics describes satisfactorily slow-motion phenomena at all scales including cosmology, its laws are wrong for fast motion, high energy processes. Especially it is incapable of accounting for the behaviour of massless fields such as electromagnetism.

After Rømer's discovery of the finiteness of the speed of light in 1676 and Bradley's discovery of aberration in 1728 it was natural to assume that light propagates in vacuo with the speed $c \approx 3 \times 10^{10}$ cm sec$^{-1}$ along straight lines with respect to some nonrotating frame of reference which coincides, at least roughly, with the centre-of-mass frame of the solar system. This hypothesis singles out a preferred ether frame, since the only transformations of the kinematical group $\mathfrak{K}$ which preserve the assumed law of light propagation are those of the elementary group $\mathfrak{E}$. One is thus led back to the original Newtonian spacetime. It is well known that neither this theory of a rigid Maxwell-Lorentz ether nor theories with deformable ethers nor emission theories of the Ritz type have been able to give a satisfactory account of the many phenomena of optics and electrodynamics of moving bodies. (For relevant experiments, see, e.g., refs. 35, 36.)

In his famous paper of 1905 on the electrodynamics of moving bodies [3] Einstein showed how the difficulties can be overcome by discarding the empirically unfounded assumption of an absolute time, and adopting instead a principle of relativity for mechanical and electromagnetic processes and by assuming the independence of the velocity of light on the velocity of the source (now experimentally established with an accuracy of $10^{-4}$, see ref. 35). Three years later his former teacher Minkowski found a geometrical characterization of the new kinematics [4]. According to him, spacetime is a pseudo-Euclidean, four-dimensional space whose metric tensor $\eta_{ab}$ has signature $(+ + + -)$. The null cones defined by $\eta_{ab}$ describe light propagation (in vacuo), the timelike straight lines represent the world lines of free particles, and the arc length

$$\int \sqrt{-\eta_{ab}\, dx^a\, dx^b} = \int \sqrt{1 - v^2}\, dt \tag{3.1}$$

($x^4 = t$, $c = 1$) of a timelike curve $L$ gives the proper time measured by a standard clock carried by a particle with world line $L$. (See Fig. 5.)

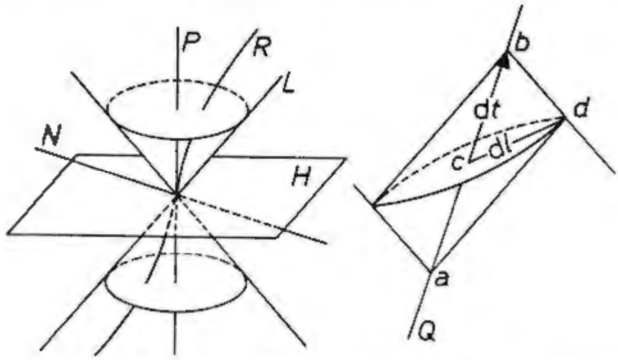


Fig. 5. Minkowskian Spacetime. $P$, $Q$ represent free particles; $R$, an accelerated particle; $L$, a light ray; $N$ a spacelike straight line. The events in $H$ are simultaneous for $P$, but $c$ and $d$ are simultaneous for $Q$. The affine structure is as in Fig. 3, but the metric is different. $dl = c\,dt$, and $b$ and $d$ have "distance" zero.

The spacetime geometry of the special theory of relativity can be derived from the assumption that there exist coordinates $x^a$ with the following two properties:

($a$) Two events $x$, $y$ can be connected by a light signal in empty space if and only if

$$(\mathbf{x} - \mathbf{y})^2 = (x^4 - y^4)^2. \tag{3.2}$$

($b$) Free particles obey the law of inertia

$$\frac{d^2 x^\lambda}{(dx^4)^2} = 0. \tag{3.3}$$

The transformations leaving these two laws invariant are precisely those which map the differential form $\eta_{ab}\, dx^a dx^b = d\mathbf{x}^2 - (dx^4)^2$ into a constant multiple of itself; they form the Poincaré group $\mathfrak{P}$, augmented by dilations. Postulate ($a$) assigns a conformal structure (a field of null cones) to spacetime, and ($b$) gives it a projective structure (a family of "straight" lines). These two primitive structures together define a Minkowskian geometry (Weyl [37] 1923, see also Fock [38]). This characterization of Minkowski space is a local one; (3.2) and (3.3) need to hold only in finite coordinate domains. If one assumes the coordinates $x^a$ to range over the whole space $\mathbb{R}^4$ one can even dispense with ($b$), but such a global requirement seems physically unreasonable. Many other approaches are known, see ref. 36.

The metric of Minkowskian spacetime is simpler than that of the nonrelativistic spacetimes since it is specified by a single tensor $\eta_{ab}$ rather than by two quantities $t$, $\mathrm{d}l^2$.

The most important difference between the nonrelativistic spacetimes and that of special relativity lies in their causal structures. In Minkowski's spacetime the causal future (past) of an event $e$ is bounded by the future (past) null cone, and thus there is a four-dimensional region whose events are causally disconnected with $e$, in contrast to the situation in nonrelativistic spacetimes. A bijection of Minkowski spacetime onto itself which preserves the causal order is the product of a dilation and an orthochronous Poincaré transformation (Zeeman, 1964 [39]).

The (coordinate) topology of special relativistic spacetime can easily be obtained from its chronological order. Let $b$ be called later than $a$, for $a$, $b \in M$, if $b$ is contained in the interior of the future null cone of $a$, written $a < b$. Then the sets $\{x | a < x < b,\ a,\ b \in M\}$ generate the topology of $M$. This way of introducing the topology of $M$, due to Alexandrow [40], is physically very satisfactory since it says that an event $x$ is close to $y$ if there is a particle $P$ through $a$ and a "short" time interval on $P$ containing $a$ within which $P$ can "communicate" with $b$. (See Fig. 6.)

The corresponding construction does not work in nonrelativistic spacetime, since it would lead to a non-Hausdorff topology. This illustrates the fundamental difference between the causal structures of relativistic and nonrelativistic spacetimes.

The absence of an observer-independent, transitive simultaneity relation between events in special (and general) relativity theory (spacelike separation is not transitive! ) implies that the ontological conception of an external world evolving "in time" has no formal counterpart in the laws of the theory. This recognition poses philosophical questions concerning the

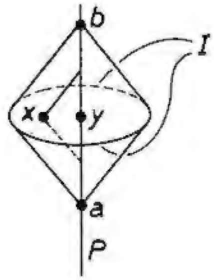


Fig. 6. Causal topology. $I$ is the causal interval between $a$ and $b$. It is a neighbourhood of $y$ containing events like $x$ causally related to a particle like $P$.

nature of time. (Weyl says [41]: The objective world simply *is*, it does not *happen.*)

Apart from the classical optical and electrodynamical effects which led to the special theory of relativity there are numerous other kinds of experimental results which confirm it. Examples are the time-dilation, now established by means of measurements of lifetimes of muons in storage rings with an accuracy of $2 \times 10^{-2}$ (ref. 42), which supports directly the validity of Equation (3.1) and therefore the existence of a pseudo-Euclidean metric, and the multitude of data on high energy collisions which could hardly be ordered reasonably without use of the metric $\eta_{ab}$ in 4-momentum space. Less direct, but at least equally convincing successes of the theory resulted from its combination with quantum theory. Examples are Dirac's electron-positron theory, the spin-statistics theorem, the CPT-theorem, the (already mentioned) classification of particles, an the Lamb shift.

There can be no doubt, therefore, that the Einstein-Minkowski spacetime theory is very nearly correct at the laboratory, atomic and nuclear scale. More precisely, one can say that the existence of a Minkowskian metric at each spacetime point is well supported empirically. On the other hand, the facts referred to do not demand the existence of global inertial frames [43]; they do not even permit one to decide whether a frame attached to the Earth's surface or one attached to a freely falling test particle is a better candidate for such a frame. Hence, the question arises how this uncertainty can be removed on observational grounds.

A related objection to special relativity theory is that its foundations involve in an essential way the law of inertia –its linear structure is based on it, and so the arguments concerned with the inseparability of inertia and gravity discussed in Section 2 all apply. These former considerations suggest that special relativity may be correct only approximately, as long as inhomogeneous gravitational fields are disregarded, and that the inclusion of gravity requires a modification of Minkowskian geometry similar to the one which led from special-nonrelativistic spacetime to general-nonrelativistic spacetime. Thus one is led to look for a theory which agrees locally (approximately) with special relativity, but has, instead of the integrable, affine connection of Minkowski spacetime, a non-integrable linear connection capable of representing, in the manner discussed above, the combined inertial-gravitational field.

In order to satisfy the first requirement, the metric should be related to the connection such that local inertial coordinate systems exist in which, at an event,

(*a*) the components of the metric tensor have their special relativistic standard values $\eta_{ab}$ = diag. $(1, 1, 1, -1)$,

(*b*) the first derivatives of the metric components vanish,
(*c*) the components of the connection vanish.

These requirements lead uniquely to the conclusion: Spacetime is a smooth 4-manifold with a pseudo-Riemannian metric $g_{ab}$ of signature ( + + + −). Its null geodesics represent light rays, the timelike geodesics of the Riemannian connection $\Gamma^a_{bc}$ associated with $g_{ab}$ represent worldlines of freely falling test particles, and the arc length along timelike lines measures the proper time shown by a standard clock. (See Fig. 7.) These assertions form the kinematical basis of Einstein's general theory of relativity [6]. The spacetime thus obtained incorporates the empirically supported structural elements of the "Newtonian" gravity theory (non-integrable connection, curvature ≡ tidal field) and of special relativity (null cones, Minkowskian inner product of 4-vectors) and it does not contain the ill-founded, absolute, too special structures (absolute time, integrable connection) of the

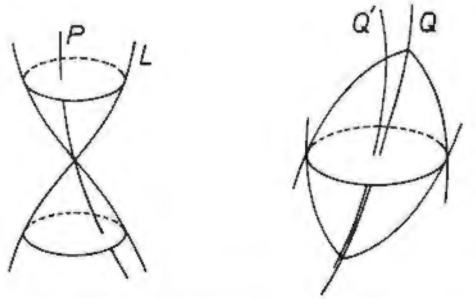


Fig. 7. Curved pseudo-Riemannian spacetime. *P, Q, Q'* represent freely falling particles; *L*, a light ray. The relative acceleration between *Q* and *Q'* indicates curvature. The affine structure is qualitatively like that in Fig. 4, the metric (causal) structure is infinitesimally as in Fig. 5. Smooth deformation of Fig. 5 gives something like Fig. 6, though different details are shown.

earlier theories. The theory admits local inertial frames as defined above, and hence it is meaningful – as in the corresponding nonrelativistic case considered in Section 2– to apply within its framework the following strong principle of equivalence [44]: For each class of physical phenomena except gravity a set of basic, local laws exists which, if expressed in terms of inertial coordinates at an event *e*, take on some standard form, independent of the spacetime location and the gravitational field gradient (the curvature). This principle is somewhat ambiguous like the correspondence principle between classical and quantum mechanics, since it does not tell which laws remain unchanged, but it is nevertheless useful for generalizing tentatively laws known in the absence of gravity to the case where gravity is present. (The ambiguity arises whenever second or higher derivatives occur, since the corresponding covariant derivatives do not commute – this again is formally similar to the factor ordering ambiguity in the quantum case.) It is the gravitational analog of the principle of minimal electromagnetic coupling.

In spite of the fundamental nature of the weak principle of equivalence it is desirable to give additional empirical reasons for abandoning flat spacetime in favour of a curved one. Two such reasons are:

(1) The terrestrial redshift measurements by Pound, Rebka and Snider (see ref. 35) are incompatible with the assumption that nuclear clocks show Minkowskian proper time, as shown convincingly by Schild [45]. Also, these experiments show that the frames of reference in which Maxwell's equations for the propagation of photons hold locally coincide, at least to within 1%, with frames falling freely towards the Earth [35]. These frames, however, are relatively accelerated, and hence cannot be considered as strict inertial frames in the sense of special relativity. (In addition, these experiments support the strong principle of equivalence, since they show that the mechanically preferred frames are also electromagnetically preferred.)

(2) The deflection of light by the solar gravitational field, whose value is now established with about 10% accuracy [35], is incompatible with a Minkowskian light cone structure. It excludes conformally flat spacetime metrics (like that of Nordström's theory). The same conclusion can be drawn from the radar time delay measurements, the "fourth test" of general relativity [35].

The preceding arguments demonstrate: If a pseudo-Riemannian spacetime metric $g_{ab}$ is assumed to exist which is observable either by means of proper times as measured by atomic or nuclear clocks, or by means of its timelike geodesics as free fall orbits of test particles, or through its null geodesics as light rays, then the curvature associated with that metric does not vanish and provides an observer-independent measure of the gravitational tidal field. Moreover, a single such metric $g_{ab}$ (together with its associated linear connection and curvature) accounts, in connection with Einstein's field equation, for the various observable phenomena; this is significant since, at least in principle, a metric is already determined by proper time measurements or by observations of geodesics separately. In view of these facts, the assertion that spacetime is "really" curved, which was already fairly well established shortly after Einstein had proposed his theory, can now, in view of the recent experimental work [35] and further theoretical analyses, be considered as well established, and in the author's opinion the phenomenological foundation for the assignment of a curved, pseudo-Riemannian structure to spacetime is as firm as those of other fundamental theoretical conceptions of physics. (The precise form of the gravitational field equation is not as firmly established empirically. This question will not be discussed here, since it concerns not so much the structure of spacetime itself, but rather the detailed nature of the coupling of the gravitational field to its sources.)

In view of the successes of special relativity in elementary particle physics one may nevertheless ask whether it is possible to incorporate also gravity into it, at the unavoidable cost of sacrificing the observability of the flat metric. If one attempts to describe gravity as a Poincaré-invariant field which has to contain a massless spin two part as an essential ingredient because of its macroscopic, long range, attractive character and its universal coupling to all other fields, then according to Kraichnan 1955, Feynman 1956, Thirring 1959, Wyss 1965, Deser 1970 (for references see Deser [46]) the resulting theory can be reformulated in such a way that only a curved metric, and not the originally postulated flat metric, occurs in the laws of the theory, which in the pure spin two case turns out to be identical to Einstein's general theory, at least at the classical level. (A consistent quantum version is not known.) The originally postulated Poincaré invariance thus turns out to be physically meaningless in the theory finally obtained, just like the flat metric which is not only unobservable, but cannot even be uniquely computed from observable quantities and does not play any useful role in the theory. (The experimentally required approximate local validity of special relativity is guaranteed by the Riemannian nature of the observable metric, and has nothing to do with the initially postulated flat metric.) In the opinion of the author these remarkable results indicate strongly that there is no satisfactory flat space theory of gravity, and they strengthen the conclusion reached in the preceding paragraph. To interpret these results as showing that Einstein's theory may as well be considered as a somewhat peculiar Poincaré invariant theory with a complicated gauge group seems (to me) inappropriate and misleading. The usefulness of the formally Poincaré invariant description of Einstein's and similar theories of spacetime and gravity for making special relativistic techniques available to them, for comparing Einstein's with other theories, and for relating it to quantum field theories is an entirely different matter not to be confused with the issue with which we are concerned here. The physicist's conception of spacetime has been changed profoundly in the transition from special relativity to general relativity, and a return to the earlier, narrower scheme is as improbable as a return from quantum to classical mechanics.

## 4. Remarks about recent work on spacetime structure; problems

Since the curved, pseudo-Riemannian manifold of Einstein seems to be the most adequate and comprehensive model of spacetime presently available, one may wish to give a physically plausible axiomatics for it. The axiomatic construction should in particular clarify why a Riemannian rather than a different kind of geometry (e.g., a Finsler geometry or a Kähler manifold) is adopted, and it should enable one to understand why the same functions $g_{ab}$ which describe the clock readings also determine the paths of test particles and light rays.

Such an approach has recently been elaborated by Pirani, Schild and the present author [47]. This work is closely related in spirit and partly inspired by papers or remarks due to Castagnino [48], Geroch, Hoffmann [51], Kronheimer [48], Kundt [51], Marzke [50], Penrose [48], Reichenbach [48], Synge [48], Trautman [48], Weyl [48], Wheeler [50], Woodhouse [48] and others. The main ideas of this approach, without technical details, will now be reviewed.

Neither rods nor clocks are used as primitive concepts. Instead, light rays and freely falling test particles are considered as the basic tools for setting up the spacetime geometry. Accordingly, the construction starts with a set $M$, the set of events, and two families $\mathfrak{L}$ and $\mathfrak{P}$ of subsets of $M$. $\mathfrak{L}$ represents the collection of all (possible) light rays, and $\mathfrak{P}$ that of all (possible) free fall world lines, briefly called particles hereafter.

The axioms about $(M, \mathfrak{L}, \mathfrak{P})$ express essentially the following. The events constituting a particle can be distinguished by means of a real parameter which can be thought of as a nonmetric, but smoothly varying time, determined only up to smooth and smoothly invertible transformations. Events which are (intuitively speaking) close to a particle are required to be connectible to the particle by precisely two light rays. Relatively to two particles (which may also be interpreted as observers carrying arbitrary clocks), events in their vicinity can be localized and coordinatized by means of the four "times" $u$, $v$, $u'$, $v'$ at which the light rays connecting the particles with the event are emitted or received (see Fig. 8); and it is postulated that such "radar coordinate systems" assign to $M$ the structure of a differentiable manifold. Moreover, it is assumed that the set $\nu_e$ of

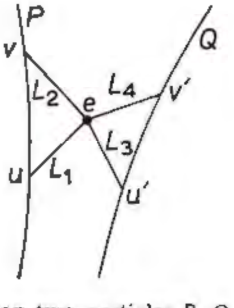


Fig. 8. Radar coordinates based on two particles $P$, $Q$ carrying clocks. $L_1$, $L_2$, $L_3$, $L_4$ are light rays.

events contained in light rays through $e$ consists, at least in an infinitesimal neighbourhood of $e$, of two (topologically) disconnected parts (which ultimately turn out to be the future and past light half cones); and that, again infinitesimally, $\nu_e$ separates the vicinity of $e$ into three parts containing, respectively, the events lying in the past, the present and the future of $e$.

A particle is assumed to be uniquely determined by an event and a direction (initial velocity) at that event, and the path-structure thus defined on $M$, which represents the combined inertial-gravitational field or, in Weyl's suggestive terminology, the guiding field, is quantitatively specified by the requirement that the law of inertia holds infinitesimally, which amounts to the assignment of a projective structure [49] to $M$. In this way the weak principle of equivalence is built into the theory.

In order to relate light propagation and free fall in accordance with experience, the set of all (possible) particles through an event $e$ is assumed to cover, again locally, the interior of the timelike region bounded by $\nu_e$ (see Fig. 9).

From these "qualitative" assumptions about light propagation and free fall which form mathematical idealizations of well-established facts and which appear to be minimal requirements for the local validity of special relativity – any departure from these would seem to be a major change of the kinematical basis of physics – it follows that there exists a unique Lorentzian conformal structure (i.e., a field of null cones derivable from a pseudo-Riemannian metric of the standard signature) on $M$ whose null geodesics are identical with the light rays. Moreover, it follows that a unique symmetric linear connection $\Gamma$ is determined by the following two requirements:

(*a*) the timelike geodesics of $\Gamma$ coincide with the free fall world lines, and

(*b*) the parallel transport of vectors defined by $\Gamma$ preserves the causal character of vectors, i.e., their being timelike, spacelike or null.

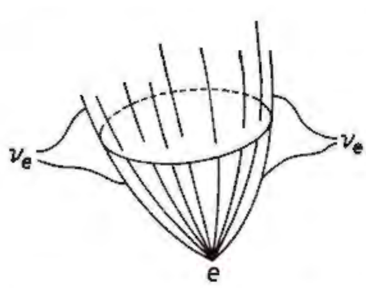


Fig. 9. The world lines of freely falling particles through $e$ fill (cover) the interior of $e$'s lightcone, $\nu_e$.

The affine parameters defined on particles by $\Gamma$ provide preferred time coordinates; appropriate clocks can be constructed by means of particles and light rays using the method of Marzke [50] or Kundt and Hoffmann [51].

The geometry thus obtained – a conformal structure and a symmetric linear connection compatible with it (in the sense of property (*b*) above)– is nothing but a Weyl geometry, invented (from a different point of view) in connection with the first unified theory of gravity and electromagnetism [52]. In the approach outlined here, this geometry appears as the natural framework for the kinematics of light rays and free particles independently of and more basic than a metric. This geometry is, in general, not (pseudo) Riemannian; in it the transport of time intervals (or distances) will in general be path-dependent. Only if one adds the assumption that time-transport be path-independent – which one has to do if one identifies gravitational time (as given by $\Gamma$) with atomic or nuclear time, since the latter is integrably transported because of the indistinguishability of particles of a particular kind – then one obtains a metric and thus arrives at the standard spacetime structure of relativity theory [53]. By adding field equations relating the curvature of $\Gamma$ to material sources, or by strengthening the infinitesimal law of inertia to the traditional, global law of inertia, one can finally specialize the theory to the general or the special theory of relativity, respectively. As discussed earlier, experience clearly decides in favour of the first possibility.

This approach shows how quantitative measures of time, angle and distance, and a procedure of parallel displacement (and hence covariant differentiation needed for formulating field equations) can be obtained constructively from "geometry-free" assumptions about light-rays and freely falling particles; pseudo-Riemannian (or Weylian) geometry is recognized even more clearly than before as the appropriate language for a generalized kinematics which allows for the unavoidable and ever-present "distortions" called gravitational fields.

A completely different, rather abstract approach to spacetime structure, the spinor approach of R. Penrose [54], will now be considered briefly. It is based on the observation that the covering group $SL_2(C)$ of the (homogeneous) Lorenz group $\mathfrak{L}$ is algebraically a much simpler object than $\mathfrak{L}$ itself, and that the simplest building blocks out of which the values of all tensor and spinor fields of standard field theory can be constructed are two-component spinors. Therefore, it is suggested to consider spacetime primarily as the carrier of such spinor fields, and to infer its structure from this its role.

Translated into mathematical language, this means that spacetime $M$ is the base of a fibre bundle $B$ whose typical fibre consists of a pair $(S, \bar{S})$ of complex, 2-dimensional vector spaces each equipped with a symplectic form (spin "metric"), and related by an anti-isomorphism $S \to \bar{S}$. Such a pair of spin spaces determines, as is well known, a real, 4-dimensional vector space $\mathfrak{R}$ $(S \otimes \bar{S})$ consisting of the hermitean spinors of the kind $t^{A\dot{A}}$ (in van der Waerden's notation), and this is the lowest-dimensional real vector space which can be built from $(S, \bar{S})$. Therefore, if and only if $M$ is 4-dimensional, it is possible to tie the fibres of $B$ to the base $M$ in a simple manner, viz., by identifying smoothly the spaces $\mathfrak{R}$ $(S \otimes \bar{S})$ with the tangent spaces. If this is done, $M$ acquires a pseudo-Riemannian structure; thus one obtains not only Einstein's spacetime, but in addition a spinor structure – which is anyhow needed since there are Fermions in nature – and hence a time and space orientation [55] in accordance with the symmetry violations observed in weak interactions [56].

The strength of this approach is that it is adapted specifically to dim $M$ = 4 and signature $(g_{ab}) = (+ + + -)$ (or $(- - - +)$, which is equivalent), rather than to another type of semi-Riemannian manifold, and that it exploits the power and simplicity of spinor calculus for the analysis of spacetime. On the other hand, one would perhaps wish a more detailed physical motivation of the choice of the ingredients $SL_2(C)$; $S, \bar{S}$; and $\sigma$: $(S \otimes \bar{S}) \rightarrow T(M)$ on which this construction is based.

Whereas the preceding two approaches are conservative in that they analyze or reconstruct the pseudo-Riemannian spacetime of standard general relativity theory, there have also been many attempts to change that structure. One attractive possibility, proposed by Cartan[21] and elaborated by several authors, enriches the geometry by a torsion tensor coupled to the spin density of matter[57]. Other theories, based, e.g., on asymmetric connections or "metrics", mostly created with the aim of obtaining unified field theories [58], will not be reviewed here. It appears that they have not led to significant physical insights, and they did not influence the main stream of physical thinking.

The great open question related to spacetime structure is its role in microphysics. In atomic, nuclear and particle physics the classical, nonrelativistic or relativistic, special spacetime structure has so far been used as if it were a universal classical external field; it even appears that the definitions of particles and fields presuppose a classical geometry with an isometry group such as $\mathfrak{G}$ or $\mathfrak{P}$. This does not fit with the claim of Einstein's general theory that the quantities defining the geometry, like $g_{ab}$ or $\Gamma^a_{bc}$, are dynamical fields, an idea which in itself is overwhelmingly convincing. Why should "geometry" stand apart from the rest of physics? "The metric" is just a particular long-range field coupled in a universal way to all carriers of energy-momentum, and it so happens that this field governs at the laboratory scale, in conjunction with electrodynamic and quantum laws, those properties of solids which we use to describe in terms of distance, congruence, etc., i.e. in terms of geometry; and at large scales this same field shows up in the phenomena called gravitation. This conception would seem to be much more satisfactory from the viewpoint of the unity of physics than one which treats "geometry" differently from "physical" structures –if only this view could be carried out in a consistent theory. So far, it has not; and it seems to the author that at present there is little hope that it will be, in the foreseeable future.

A crucial question related to this deep problems is: At which scale (if at all) does the concept of a smooth spacetime manifold cease to be adequate in the small? Does this happen at $10^{-13}$ cm, at $10^{-33}$ cm, or when? That it does happen at some scale is to be expected, in view of the (from a physicist's standpoint) highly artificial, non-constructive nature of the continuum of real numbers [59]; and because of a breakdown of the localizability of particles connected with creation processes [60]; or in view of violent quantum fluctuations of $g_{ab}$ and $\Gamma^a_{bc}$ at small scales [11]; it is also indicated by a number of singularity theorems established during the last several years [61].

What kind of "space" should take the place of the smooth manifold? A geometry of lumps without smallest elements (points), a statistical geometry as suggested by Menger [62], or a collection of classical spacetimes or three-spaces, weighted by probability amplitudes (see Wheeler[11]), or something else? "The difficulty is that all our present theoretical work is based on a microscopic continuum and one is faced by the rather formidable problem of re-doing all physics in a continuum-free manner", says Finkelstein in this context [63]. One generalization of ordinary spacetimes is provided by the causal spaces of Kronheimer and Penrose [48], another one is the "quantum-computer-geometry" of Finkelstein [63], but so far little is known about their physical potentialities. Here we have arrived at the border of spacetime knowledge, and are left with questions only.

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